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COMMENT

The superfluid phase transition in pulsars

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Abstract. Renormalisation group equations are derived from the Ginzburg–Landau free energy for p-wave paired neutron-star matter in the presence of large magnetic fields. The effect of fluctuations on the nature of the superfluid phase transition is discussed.

Neutron star matter in the density range $1\text{–}8\times10^{14}\ \mathrm{g\,cm^{-3}}$ is believed to be a p-wave superfluid like ^{3}He, but with a strong spin-orbit or tensor force producing pairing in a $J=2$ state (Takatsuka 1972, Hoffberg *et al* 1970 and references therein). In an earlier paper (Bailin *et al* 1979) it was shown that the transition from the normal to the superfluid phase of neutron star matter is first-order when account is taken of fluctuations by renormalisation group methods. In the present paper we extend this work to the case where large magnetic fields such as arise in a pulsar are present.

In the absence of magnetic fields, the order parameter for neutron star superfluidity is a complex 3×3 matrix A_{ij} which, because of the $J=2$ nature of the pairing, is traceless and symmetric. The Ginzburg–Landau bulk free energy density is of the form

$$F_{\mathrm{B}}=r\,\mathrm{Tr}(AA^*)+\tfrac{1}{4}u|\mathrm{Tr}\,A^2|^2+\tfrac{1}{4}v(\mathrm{Tr}\,AA^*)^2+\tfrac{1}{4}w\,\mathrm{Tr}\,A^2A^{*2} \tag{1}$$

(Sauls and Serene 1978), where r vanishes at the transition temperature in the mean field approximation. The most general bending (or strain) free energy consistent with symmetry under simultaneous spin and space rotations, and with phase symmetry, is

$$F_{\mathrm{s}}=\alpha^{-1}\,\partial_iA^*_{ki}\,\partial_jA_{kj}+\epsilon_{ijk}\,\partial_jA^*_{pk}\epsilon_{ilm}\,\partial_lA_{pm} \tag{2}$$

apart from total divergences.

A magnetic field adds two terms to the free energy functional (Ambegaokar and Mermin 1973, Muzikar *et al* 1979),

$$F_{H1}=\mathrm{i}\eta\epsilon_{j\mu\nu}h_jA_{\mu p}A^*_{\nu p} \tag{3}$$

and

$$F_{H2}=\xi h_\mu A^*_{\mu i}A_{\nu i}h_\nu \tag{4}$$

where

$$h_i=\gamma H_i/2k_{\mathrm{B}}T_{\mathrm{c}} \tag{5}$$

with $\gamma\sigma/2$ the magnetic moment of a neutron and H_i the components of the magnetic field. The coefficients in F_{H1} and F_{H2} have the values

$$\eta=\eta'\xi_{\mathrm{T}}^{-2} \tag{6}$$

and

$$\xi=(7\zeta(3)/4\pi^2)\xi_{\mathrm{T}}^{-2} \tag{7}$$

where η' is the Ambegaokar–Mermin parameter, and $\xi_T^2 = 3\xi_0^2/5$. (For details of the form of the free energy functional used, see Jones *et al* 1976.) Ambegaokar and Mermin (1973) obtained a value for the parameter η' from the splitting of the A and A_1 phases of ^{3}He. This value must be modified to allow for the ratio of the superfluid critical temperatures for neutron star matter and ^{3}He and for the ratio of the neutron and ^{3}He nuclear magnetic moments. We then find that F_{H1} and F_{H2} are comparable in magnitude for magnetic fields of the order of 10^{14} G. Estimates of the magnetic field in the interior of a neutron star range from 10^{12} to 10^{16} G (Beckenstein and Oron 1979 and references therein). We therefore discuss both the case $H > 10^{14}$ G when F_{H2} is dominant and $H < 10^{14}$ G when F_{H1} is dominant. The critical region for the superfluid phase transition in neutron star matter has been estimated to be (Bailin *et al* 1979)

$$(T_c - T)/T_c \lesssim 10^{-5} - 10^{-4} \tag{8}$$

though the value is very sensitive to T_c/T_F. The value of equation (8) implies that when $H > 10^{14}$ G the effect of the magnetic field will be important throughout the whole critical region, whereas when $H < 10^{14}$ G it will only be important in a secondary critical region sufficiently close to T_c. (It is coincidental that 10^{14} G is also the magnetic field at which F_{H1} and F_{H2} are comparable.) Throughout we shall assume a uniform magnetic field along the z-direction, or, in the case of d dimensions of space, along the d-direction.

We consider first the case $H > 10^{14}$ G. To find which components of the order parameter are relevant to the phase transition, we must diagonalise the quadratic terms in the bulk free energy with the inclusion of the magnetic field term F_{H2} and look for soft modes. The result is an order parameter which is a traceless symmetric 2×2 matrix. For renormalisation group calculations we have to work in $d = 4 - \epsilon$ dimensions of space. With the gradient terms of equation (2), the order parameter in zero magnetic field is then a $d \times d$ matrix and, after taking account of F_{H2}, we are left with a traceless symmetric $(d-1)\times(d-1)$ matrix. We construct renormalisation group equations in the form given by 't Hooft (1973), and the calculations are greatly simplified by the fact that we may use massless propagators ($r = 0$). In momentum space, the inverse propagator arising from equation (2) is then of the form

$$P^{-1}_{ki,lj}(q) = \delta_{ki}[(\alpha^{-1} - 1)q_l q_j + (q^2 + cq_d^2)\delta_{lj}] \tag{9}$$

where k, i, l and $j = 1, \ldots, d-1$, and $q^2 = q_1^2 + \ldots + q_d^2$. By allowing c to be a free parameter, we obtain the most general inverse propagator consistent with the reduced symmetry in the presence of the magnetic field. In constructing the propagator we must symmetrise and subtract traces to avoid spurious contributions from other than $J = 2$ intermediate states. Correct to linear order in

$$\lambda \equiv \alpha - 1, \tag{10}$$

the propagator has the form

$$\begin{aligned}(q^2 + cq_d^2)^2 P_{ki,lj}(q) &= (q^2 + cq_d^2)\left(\tfrac{1}{2}(\delta_{ki}\delta_{lj} + \delta_{kj}\delta_{li}) - \frac{1}{(d-1)}\delta_{kl}\delta_{ij}\right) \\ &\quad + \lambda\Big(\tfrac{1}{4}(\delta_{ki}q_l q_j + \delta_{lj}q_k q_i + \delta_{kj}q_l q_i + \delta_{li}q_k q_j) \\ &\quad - \frac{1}{(d-1)}(\delta_{kl}q_i q_j + \delta_{ij}q_k q_l) + \frac{(q^2 - q_d^2)}{(d-1)^2}\delta_{kl}\delta_{ij}\Big)\end{aligned} \tag{11}$$

where again k, i, l and $j = 1, \ldots, d-1$ and $q^2 = q_1^2 + \ldots + q_d^2$.

An order-ϵ^2 two-loop calculation gives the Callan–Symanzik functions (linearised about $\lambda = c = 0$)

$$(S/2)^{-2}\beta_\lambda = \lambda\left(\tfrac{5}{3}u^2 + \tfrac{5}{6}v^2 + \tfrac{67}{288}w^2 + \tfrac{13}{18}uw + \tfrac{17}{36}vw\right) \tag{12}$$

and

$$(S/2)^{-2}\beta_c = -\tfrac{1}{3}\lambda\left(\tfrac{5}{3}u^2 + \tfrac{5}{6}v^2 + \tfrac{67}{288}w^2 + \tfrac{13}{18}uw + \tfrac{17}{36}vw\right) \tag{13}$$

where

$$S/2 = \pi^{d/2}/(2\pi)^d\Gamma(d/2). \tag{14}$$

Since the coefficient of λ in equation (12) is positive definite, λ has an infrared stable fixed point at $\lambda = 0$. Equation (13) then shows that at this fixed point c is a marginal variable. Accordingly we shall take $\alpha = 1$ in subsequent calculations, but we must allow c to be non-zero. When $\alpha = 1$, the propagator simplifies to a scalar meson propagator, and in general we may take A_{ij} to be an $n \times n$ matrix with n different from $d-1$. The only effect of c being non-zero is everywhere to replace the geometrical factor $S/2$ by the geometrical factor

$$\frac{S'}{2} = \frac{S/2}{B[\frac{1}{2}(d-1), \frac{1}{2}]}\int_0^\pi \frac{\sin^{d-2}\theta \, d\theta}{(1 + c\cos^2\theta)^2}. \tag{15}$$

Absorbing a factor of S'/S into the definitions of u, v and w, the Callan–Symanzik functions for u, v and w are exactly as in equation (7) of Bailin *et al* (1979). Since we may regard the order parameter either as a 2×2 matrix or as a $(d-1)\times(d-1)$ matrix with $d = 4-\epsilon$, it is not clear whether to take $n = 2$ or $n = 3$. However, in either case there are no infrared stable fixed points, and we expect a first-order phase transition. (The stability of the symmetric fixed point $u^* = w^* = 0$, $\frac{1}{2}S'v^* = 2\epsilon/(n^2 + n - 2 + 8)$ is marginal for $n = 2$ in an order-ϵ calculation. However, a general result of Brezin *et al* (1974), at order ϵ^2, shows that it is unstable for more than $4 - 2\epsilon$ real fields, and for $n = 2$ we have four real fields.)

Finally, we mention the case $H < 10^{14}$ G in which F_{H1} is dominant. Diagonalising the mass matrix and looking for soft modes in this case shows that there is only a single complex field relevant to the phase transition. The propagator is then an asymmetric scalar meson propagator of the type discussed by Chang and Stanley (1973) and Grover (1973). Since this type of asymmetry has no effect on the critical behaviour, the phase transition is second-order (with critical exponents characteristic of two real fields). As observed earlier, for $H < 10^{14}$ G the magnetic field does not control the whole critical region, but only a secondary critical region close to T_c.

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References

Ambegaokar V and Mermin N D 1973 *Phys. Rev. Lett.* **30** 81
Beckenstein J D and Oron E 1979 *Phys. Rev.* D **19** 2827

Bailin D, Green R J and Love A 1979 *J. Phys. A: Math. Gen.* **12** L147
Brezin E, Le Guillou J C and Zinn-Justin J 1974 *Phys. Rev.* B **10** 892
Chang T S and Stanley H E 1973 *Phys. Rev.* B **8** 4435
Grover M K 1973 *Phys. Lett.* **44A** 253
Hoffberg M, Glassgold A E, Richardson R W and Ruderman M 1970 *Phys. Rev. Lett.* **24** 775
't Hooft G 1973 *Nucl. Phys.* B **61** 455
Jones D R T, Love A and Moore M A 1976 *J. Phys. C: Solid St. Phys.* **9** 743
Muzikar P, Sauls J A and Serene J W 1979 *Preprint*
Sauls J A and Serene J W 1978 *Phys. Rev.* D **17** 1524
Takatsuka T 1972 *Prog. Theor. Phys.* **48** 1517